Exercise 26

Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$g(t) = \frac{1}{\sqrt{t}}$$

Solution

Calculate the derivative of $g(t)$ using the definition.

$$\begin{aligned}
g'(t) &= \lim_{h\to 0} \frac{g(t+h) - g(t)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} \\
&= \lim_{h\to 0} \frac{\frac{\sqrt{t}}{\sqrt{t}\sqrt{t+h}} - \frac{\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}}{h} \\
&= \lim_{h\to 0} \frac{\frac{\sqrt{t}-\sqrt{t+h}}{\sqrt{t}\sqrt{t+h}}}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}} \\
&= \lim_{h\to 0} \frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t}\sqrt{t+h}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \\
&= \lim_{h\to 0} \frac{\left(\sqrt{t} - \sqrt{t+h}\right)\left(\sqrt{t} + \sqrt{t+h}\right)}{h\sqrt{t}\sqrt{t+h}\left(\sqrt{t} + \sqrt{t+h}\right)} \\
&= \lim_{h\to 0} \frac{(t) - (t+h)}{h\sqrt{t}\sqrt{t+h}\left(\sqrt{t} + \sqrt{t+h}\right)} \\
&= \lim_{h\to 0} \frac{-h}{h\sqrt{t}\sqrt{t+h}\left(\sqrt{t} + \sqrt{t+h}\right)} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{t}\sqrt{t+h}\left(\sqrt{t} + \sqrt{t+h}\right)} \\
&= \frac{-1}{\sqrt{t}\sqrt{t}\left(\sqrt{t} + \sqrt{t}\right)} \\
&= -\frac{1}{t(2\sqrt{t})} \\
&= -\frac{1}{2t^{3/2}}
\end{aligned}$$

The domain of $g(t)$ is $\{t \mid 0 < t < \infty\}$, and the domain of $g'(t)$ is $\{t \mid 0 < t < \infty\}$.